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ABSTRACT

The so-called discrete approach in calculus instruction involves introducing topics from the calculus of finite differences and finite sums, both for motivation and as useful tools for applications of the calculus. In particular, it provides an ideal setting in which to incorporate computers into calculus courses. This approach has been successfully implemented on an experimental basis in both traditional calculus sequences for math, science, and engineering majors, and in applied calculus for business majors. A detailed description of the approach is included in this report on the actual outcomes of these courses. (Author/CHV)

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COMPUTER-ORIENTED CALCULUS COURSES

USING FINITE DIFFERENCES

by

Sheldon P. Gordon

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Computer-Oriented Calculus Courses Using Finite Differences[1]

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Introduction

At the Seventh Conference on Computers in the Undergraduate Curricula, the author presented a paper describing a new approach to teaching calculus. This so-called *discrete approach* involves introducing topics from the calculus of finite differences and finite sums both for motivation and as exceptionally useful tools for applications of the calculus. In particular, this approach provides an ideal setting in which to incorporate computers into calculus courses in an extremely natural way. The discrete approach and the class materials prepared by the author based on it have been used in several different calculus courses directed towards different audiences. They have been implemented within the traditional calculus sequence for math, science, and engineering majors at Suffolk County Community College and have been used in an applied calculus course for business majors at C. W. Post College.

The present paper is intended to report on the actual outcomes of these experiments. These detailed results and the accompanying analyses were not available in time for the previous paper, which consisted primarily of descriptions of the approach. In addition, the present paper will indicate how the discrete approach can be fruitfully used in virtually every type of calculus course offered today, not just in the non-science oriented course discussed in the previous paper.

As was pointed out in the earlier paper, there are a number of major benefits made possible by using the discrete approach. First, it provides a natural context for computer applications in the course, since all uses of the computer involve reducing continuous functions and operations to discrete analogues. Thus, the underlying mathematical concepts and techniques for computer usage will be available, rather than brought in in an often artificial way having little cohesiveness.

Second, having material on finite differences and sums available, it is a very simple matter to expand the scope of the course slightly to allow it to encompass several easy, yet useful, techniques from the field of numerical analysis. For example, approximation and interpolation methods can be covered which give the students a feeling for where all of those "given" functions really come from. In practice, no one will actually be solving significant problems involving calculus without the use of computers, and therefore an introduction to numerical methods is certainly desirable. All too few students go on to take a full course in the subject and, even for those who do, it usually stands as an isolated subject disjoint from the remainder of the mathematics courses taken in a typical undergraduate program. An early introduction to the subject within calculus should go far towards rectifying this situation.

Third, much of the current developments in applied mathematics are taking place in the non-physical sciences such as biology, economics, psychology, and so forth. In these areas, though, the majority of the mathematical models used are based on finite difference equations since the processes they reflect are inherently discrete. Consequently, the discrete approach provides the opportunity to develop some of these discrete models in a very natural way. For example, growth and

decay problems are far easier to visualize in a discrete setting and, in fact, can be nicely covered during the first week of the course. This also provides some concrete application of the subject matter very early in the course, thereby sustaining student interest. This is especially necessary in an applied calculus course where the usual month or more of purely mathematical development is likely to lose the interest of most students.

Fourth, the students are exposed to most concepts and techniques twice, first in the discrete case and again in the continuous case. As a result, this on-going review provides the students with a greater understanding of and facility with the basic notions and methods of calculus. This is particularly beneficial in many calculus courses where ever greater numbers of students are entering with extremely poor backgrounds in mathematics.

Detailed Description of the Approach

The mathematical foundations of the approach and the course are as follows. The calculus of finite differences and finite sums contains discrete analogues of most major concepts and techniques of the infinitesimal calculus without recourse to limits. For example, there are discrete parallels to such topics as maxima and minima of functions, the chain rule, the fundamental theorem of integration, techniques of integration, Taylor series expansions, and partial derivatives. (See, for example, any of the standard texts on the calculus of finite differences.) However, the absence of the limit reduces all work simply to the level of arithmetic and algebraic manipulation. Moreover, these discrete formulations are the natural ones to have available in any effort involving significant computer applications.

The idea behind the approach is to use the concepts and techniques from finite differences and sums as the underlying source of motivation for the infinitesimal calculus and the appropriate setting for solving "real" problems using discrete approximations. Specifically, most of the standard topics in calculus are taken up first in the discrete context. This leads to an easier and fuller understanding of the concepts and much greater facility with the techniques for most students in the class than is obtained in the purely continuous approach.

Furthermore, the infinitesimal calculus arises from the discrete calculus by simply passing to the limit as the step approaches zero. Thus, after the students have mastered a particular topic from the finite calculus and feel on firm ground, it is then a very simple and natural extension to introduce the limit process and thereby derive the corresponding result in the infinitesimal calculus. This natural transition has the additional advantage of presenting the students with a brief review of all concepts and methods as they are developed in the new context. Simultaneously, the students are shown how the discrete formulations can be used to approximate the continuous ones and they have the opportunity to actually apply all of these techniques on the college computer. This, in turn, fosters a still deeper understanding of the material.

The discrete approach produces a solid calculus course in which the mathematics is developed in a context which is directly oriented towards a computer philosophy. The use of the discrete foundation for the various topics provides the most natural setting for introducing the use of the computer for solving all types of problems involving calculus. Many of these areas where the computer has great usefulness, such as in evaluating limits, approximating derivatives, evaluating definite integrals, evaluating and approximating functions and so on, are already well known and need not be mentioned in any detail here. However, we will examine some of the particular applications which arise from the discrete approach and which are natural consequences of it. A course based on this approach begins with a study of sequences and the properties of the forward difference operator Δ acting on them. This is then extended to differences of an arbitrary function $y = f(x)$ defined on an interval $[a, b,]$. If a step h is then chosen, the corresponding sequence of differences

$\Delta f(a), \Delta f(a+h), \Delta f(a+2h), \ldots, \Delta f(a+nh)$ can be easily determined, especially with the aid of a computer when the function is complicated or the step is small and the number of terms is correspondingly large. By analyzing the signs of these terms, it is possible to determine intervals where the function is basically increasing or decreasing and thereby find approximate locations for the maxima or minima of the function. For instance, if $f(x) = 8x^3 - 20x^2 + 14x - 3$ on the interval [0,2] and if we choose $h = .25$, then we have the following table of differences:

$f(0) = -.3$	$\Delta f(0.) = 2.375$
$f(.25) = -.625$	$\Delta f(.25) = .625$
$f(.5) = 0.$	$\Delta f(.5) = -.375$
$f(.75) = -.375$	$\Delta f(.75) = -.625$
$f(1.) = -1.$	$\Delta f(1.) = -.125$
$f(1.25) = -1.125$	$\Delta f(1.25) = 1.125$
$f(1.5) = 0.$	$\Delta f(1.5) = 3.125$
$f(1.75) = 3.125$	$\Delta f(1.75) = 5.875$
$f(2) = 9.$	

By examining these values, we see that the function is increasing overall between $x = 0$ and $x = .5$, then is decreasing overall from $x = .5$ to $x = 1.25$, and then is again increasing overall from there to the endpoint. Consequently, we (and the students) see that there is a maximum value for the function near $x = .5$ and a minumum value near $x = 1.25$. This type of development previews the entire concept of maximizing and minimizing functions at a stage prior to the introduction of the limit, let alone the derivative. Simultaneously, this approach sets the groundwork for the notion of a computer search operation for locating the approximate values of critical points. An added advantage of this early introduction of significant problems and the techniques needed to solve them is that the student is not required to spend the usual full month or more in purely mathematical development as in the traditional calculus course. Rather, he has enough mathematics available, using finite differences, to tackle interesting and/or practical problems and to have techniques available which are virtually begging for the use of the computer in their application. As a result, this approach enables the instructor to keep the students' interest at the early stages of the course where, in the traditional course in calculus, this is not the case.

Moreover, by using techniques such as these from the calculus of finite differences, it is possible to keep the course far more cohesive in the sense that this material presents a unifying theme for the course. In this regard, once the theoretical aspects of optimization theory are treated using derivatives, the above approach can be reintroduced for a variety of applications involving computerized solutions. For one, it can be applied to a given function to locate approximate values for critical points and these approximations are then used as initial values for the Newton's Method, say, to zero in on the roots.

Alternatively, the methods can be extended to supply discrete analogues of the first and second derivative tests for maxima and minima which can be applied to the entries in the difference table. For example, in the illustration above, we find that, using Newton's Method starting respectively at $x_o = .5$ and at $x_o = 1.25$, the critical points $f(x)$ are located at $x = .5$ and at $x = 1.1666666$. It is easy to see that the first difference changes from positive to negative about $x = .5$, thus indicating a probable maximum there. Similarly, about $x = 7/6$, Δf changes from negative to positive, indicating a probable minimum at this point.

Alternatively, the second differences about $x = .5$ are negative, which also indicates a probable maximum there and similarly, about $x = 7/6$, $\Delta^2 f$ is positive, again indicating a probable minimum.

Admittedly, there are other numerical techniques which are faster, such as the bisection method to find the roots. However, the one described above fits into the calculus course in a very natural way and is also less likely to miss critical points if the step h is taken small enough. Moreover, having had such an introduction to numerical methods, the student is much more likely to take a full course in numerical analysis subsequently and to realize the interactions between that subject and other areas of mathematics.

In terms of the actual implementation of the computer aspect of the course, the students were given several hours of programming instruction spread over about two weeks of classes. During the first use of the discrete approach, we used a simplified version of FORTRAN, while we used BASIC during a second trial. The latter is certainly preferable for students who have never had programming before. However, it is this writer's belief that FORTRAN would be more desirable in the sense that it is by far the more useful language in terms of eventual computer usage in practical situations.

Initially, to give the students some confidence and a gradual introduction to the language and its capacities, they were supplied with sample programs illustrating such concepts as evaluating limits of sequences and functions. They were expected to supply their own choices of sequences and functions and other parameters and to run these programs for a variety of different cases. Throughout the course, we supplied a series of sample programs, each successive one having more "holes" which the students had to fill. The following is a list of the topics covered in this manner:

1. Terms of a sequence.
2. Growth and decay.
3. Limit of a sequence.
4. ε-N limits of sequences.
5. Differences of functions.
6. Sequential limits of functions.
7. ε-δ limits of functions.
8. Derivatives from the definition.
9. Higher derivatives via limits.
10. Search for intervals of increase and decrease.
11. Newton's method.
12. Discrete tests for max and min.

Analysis of the Results

The most significant component of this type of discussion regarding introducing additional material and techniques into an existing course must be the question of the time involved. In order to incorporate the material on both finite differences and the computer into the first term of the calculus sequence, it was necessary to devote approximately two weeks worth of class time, total. Normally, this would constitute an inordinate amount of the term. However, as indicated above, the discrete approach involves developing the usual calculus topics in the discrete case first and then carrying over in the limit to the continuous case. In each instance, we found that there is a very considerable saving in time during the development of the continuous cases, since the students were thoroughly familiar with the concepts already from the discrete setting. In fact, this is true to such an extent that the net loss in time was virtually nil. This was accompanied, though, by an overall major gain in the level of student understanding and facility, as measured by student performance on tests, student comments (both verbal and on course questionnaires) and my personal evaluation of individual classroom response.

In addition, the results of implementing the discrete approach at C. W. Post College bear this out fully. The approach was used in a one-semester applied calculus course for business majors. Considering the extreme shortness of such courses, the instructor chose to use the material solely as motivational without any direct use of the computer. As in our own experience, the loss of time involved in introducing the additional topics on finite differences was fully compensated for by the savings in developing the continuous cases. In fact, the instructor's personal opinion is that this material simplifed the calculus topics very considerably for the students and, as a result, of the ten sections which ran, this experimental section was one of the three that actually finished the entire syllabus. Most of the other sections did not have time to finish most if not all of the final chapter. (The book and syllabus have since been changed.) Moreover, despite introducing the extra material on finite differences and covering more calculus topics than most of the other sections, the overall grad average for the experimental section was virtually identical to the average department-wide, namely 2.55 versus 2.57 on a scale of 0 to 4.

In terms of the grades achieved in our experiments at Suffolk CC, complete figures are available for the spring, 1976 semester, but not yet for the fall, 1976 term. The results are summarized in the table below. The entry "No Credit" includes both failing students and those who have withdrawn during the course. The effect of combining the two drastically lowers the overall grade average in the course. Unfortunately, the breakdown is not available.

	Department-wide		*Experimental*	
	No.	%	No.	%
A	28	12	8	26
B	43	19	3	10
C	48	21	7	23
D	26	11	4	13
Incomplete	4	2	1	3
No credit	77	34	7	23

The grade average for the 227 students enrolled department-wide was 1.60 on a scale of 0 to 4, while the results for the experimental section were 2.00. Thus, the results for the experimental section are significantly better than the department-wide figures. Admittedly, this can be attributed to a variety of causes, not the least of which is the particular group of students, which we personally feel was a bright group. However, we do feel that the approach used is also a major factor in the class' success, as is indicated in the following extract from a course evaluation we conducted:

1. To what extent have the presentation and the notes contributed to your interest in and liking for the course?

Greatly - 76 percent Little - 24 percent
Turned off -0

2. If you have taken calculus before, how would you rate the presentation using differences and computers?

Much better - 60 percent
Slightly better - 20 percent
Comparable - 20 percent
Worse - 0

3. What is the overall reaction to the materials distributed?

Excellent - 75 percent Good - 25 percent
Fair - 0 Poor - 0

4. How would you rate them compared to the text (Leithold's *Calculus*)?

Far superior - 73 percent Better - 27 percent
Comparable - 0 Worse - 0

As mentioned above, detailed results of the fall, 1976 semester are not available yet. The overall average for the experimental section, however, was about 1.7, and is therefore slightly better than the departmental results for the previous semester. Our assessment, though, is that this group was not an especially distinguished one.

In conclusion, we feel that the discrete approach is an extremely valuable one and can be profitably incorporated in virtually any calculus course, especially if there is to be a computer orientation. However, it can be used separately as purely motivational or for introducing discrete models, or any combination of these facets. Specific recommendations for its usage in a variety of different types of calculus courses are given in the following table.

	Comp.	*Motiv.*	*Discr. Mods.*
Honors calculus	x		
Traditional math/science calculus	x	x	x
Slow calculus	x	x	
One-term applied calculus		x	x
Two term applied calculus	x	x	x

Note

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Reference

Gordon, Sheldon P. *A Computer Oriented Calculus Sequence for Social Science Students.* Proc. 7th Conf. on Computers in the Undergraduate Curricula, 243-246 (1976).